TRINITY COLLEGE

Year 12 Mathematics Specialist
Test 4 2019

Section 1 Calculator Free

Integration

STUDENT'S NAME _____

DATE: Monday 1 July **TIME:** 33 minutes **MARKS**: 33

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, formula page

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (5 marks)

Determine

(a) $\int \frac{3x-2}{3x^2-4x+5}\,dx$ [2]

(b) $\int \cos 2x \sin^3 2x\,dx$ [3]

2. (6 marks)

(a) Express $\frac{5x-11}{(x+2)(2x-3)}$ in the form $\frac{a}{x+2}+\frac{b}{2x-3}$ [3]

(b) Hence, determine $\int \frac{5x-11}{(x+2)(2x-3)}\,dx$ [3]

3. (9 marks)

Determine

(a) $\int \frac{x^2}{x-1}\,dx$ [4]

(b) $\int \sqrt{9-x^2}\, dx$ let $x = 3\cos\sigma$ [5]

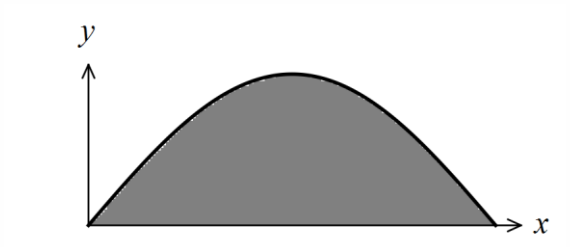
4. (7 marks)

(a) By using an appropriate trigonometric substitution, simplify in terms of u , the expression $4-x^2$ where $x=2\sin u$ [2]

(b) Hence, evaluate $\int_1^{\sqrt{3}} \frac{x}{4-x^2}\,dx$ exactly [5]

5. (6 marks)

Consider the area enclose by $\sin x$ and the x axis shown below.



(a) Determine the exact volume when the shaded area is rotated about the x axis. [3]

(b) Determine the exact volume when the shaded area is rotated about the y axis. [3]

You may find the following formulas useful:

$$\frac{d}{dx}(\sin x - x\cos x) = x\sin x$$

$$Vol_y = 2\pi\int_c^d x[f(x)]\,dx$$

TRINITY COLLEGE

Year 12 Mathematics Specialist
Test 4 2019

Section 2 Calculator Assumed

Integration

STUDENT'S NAME _____

DATE: Monday 1 July **TIME:** 17 minutes **MARKS**: 17

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, formula page

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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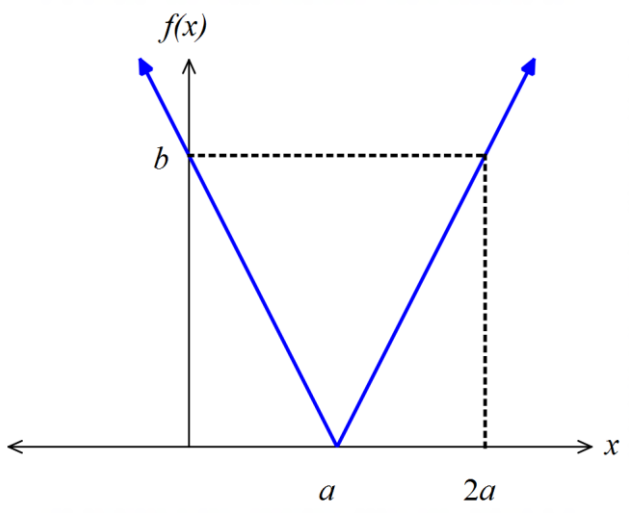
6. (8 marks)

Function f is defined by its graph shown below. The constants $a, b > 0$ where $b > a$.

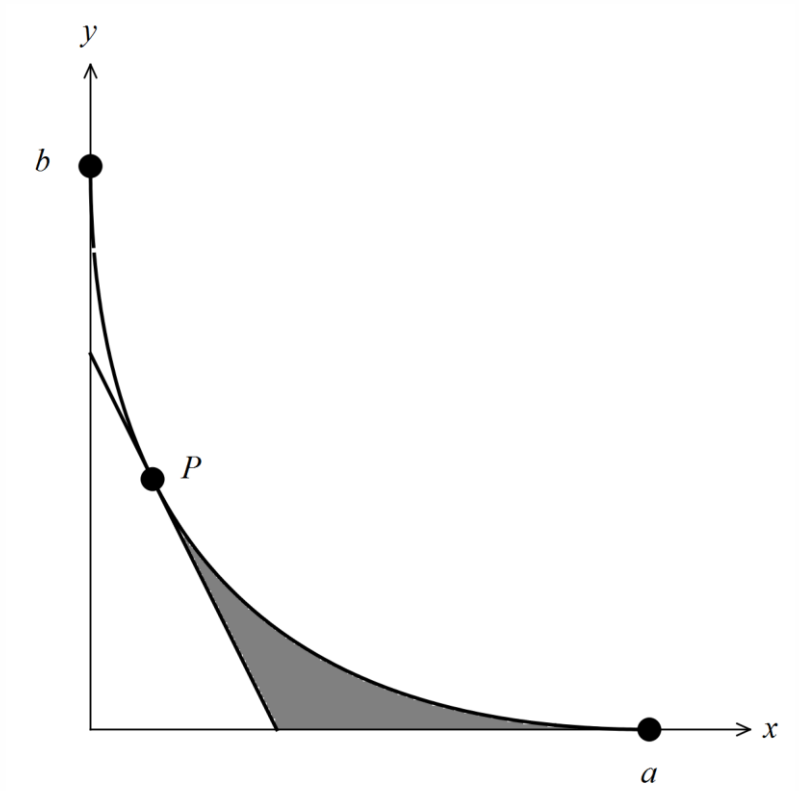


(a) Determine the defining rule for function $f(x)$ in terms of a, b . [3]

(b) By using the substitution $u = 2x - a$, determine an expression, in terms of a, b , for the value of $\int_{\frac{a}{2}}^{a} f(2x-a)dx$ [5]

7. (9 marks)

The diagram shows the curve with equation $\sqrt{x}+\sqrt{y}=3$ where points a, b are the intercepts of this curve. A tangent is drawn to the curve at point $P(1, 4)$ with equation $2x+y=6$.



The shaded area on the diagram is bounded by the curve, the tangent and the x axis.

(a) Determine the exact area of the shaded region. [5]

The shaded region is then rotated about the x axis.

(b) Calculate the volume of the resulting solid, correct to 0.01 cubic units. [4]